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15MAT11

First Semester B.E. Degree Examination, Aug./Sept.2020 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the n^{th} derivatives of $\frac{x^2 - 4x + 1}{(x + 2)(x^2 - 1)}$. (06 Marks)
- b. Find the angle of intersection between the curves $r = ae^{\theta}$ and $re^{\theta} = b$ (05 Marks)
- c. Obtain the pedal equation of the curve $r = a(1 + \cos\theta)$. (05 Marks)

OR

- 2 a. If $y = \sin^{-1}x$ prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ (06 Marks)
- b. Find the pedal equation of the curve $r^n \operatorname{cosec} n\theta = a^n$. (05 Marks)
- c. For the curve $y = \frac{ax}{a + x}$ show that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$. (05 Marks)

Module-2

- 3 a. Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$. Hence find the value of $\sin 91^\circ$ correct to four decimal places. (06 Marks)
- b. Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$ (05 Marks)
- c. If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$ (05 Marks)

OR

- 4 a. Obtain the Maclaurin's expansion of the function $\log_e(1+x)$ up to fourth degree terms and hence find $\log_e(1-x)$. (06 Marks)
- b. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ prove that $\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z} = 0$ (05 Marks)
- c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ show the $\frac{\partial(uvw)}{\partial(xyz)} = 4$. (05 Marks)

Module-3

- 5 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$. Also find the velocity and acceleration at $t = 1$ in the direction $2\hat{i} + \hat{j} + 2\hat{k}$. (06 Marks)
- b. Find the directional derivative of $f(x, y, z) = xy^3 + yz^3$ at the point $(2, -1, 1)$ in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$. (05 Marks)
- c. Find constants a and b such that $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational. (05 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8=50, will be treated as malpractice.



OR

- 6 a. Show that the vector $\vec{v} = (x + 3y)\mathbf{i} + (y - 3z)\mathbf{j} + (x - 2z)\mathbf{k}$ is a solenoidal vector. Also find $\text{Curl } \vec{v}$. (06 Marks)
- b. If $\vec{F} = (x + 3y + 1)\mathbf{i} + \mathbf{j} - (x + y)\mathbf{k}$ show that $\vec{F} \cdot \text{curl } \vec{F} = 0$ (05 Marks)
- c. Prove that $\nabla \times (\phi \vec{A}) = \phi(\nabla \times \vec{A}) + \nabla \phi \times \vec{A}$. (05 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int \sin^n x \, dx$ and evaluate $\int_0^{\pi/2} \sin^n x \, dx$. (06 Marks)
- b. Solve $(y \cos x + \sin y + y)dx + (\sin x + x \cos y + x)dy$. (05 Marks)
- c. Find the orthogonal trajectories of the family of asteroids $x^{2/3} + y^{2/3} = a^{2/3}$. (05 Marks)

OR

- 8 a. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}} dx \rightarrow 06$. (06 Marks)
- b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$ (05 Marks)
- c. A body in air at 25°C cools from 100°C to 75°C in one minute. Find the temperature of the body at the end of 3 minutes. (05 Marks)

Module-5

- 9 a. Solve the following system of equations by Gauss elimination method:
 $x + y + z = 6$
 $x - y + z = 2$
 $2x - y + 3z = 9$ (06 Marks)
- b. Use power method to find the largest eigen value and the corresponding eigen vector of the matrix A, taking $[1, 0, 0]^T$ as initial eigen vector. Perform three iterations.
$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$
 (05 Marks)
- c. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Find the inverse transformation. (05 Marks)

OR

- 10 a. Solve the following system of equations by Gauss Seidal method
 $10x + y + z = 12$
 $x + 10y + z = 12$
 $x + y + 10z = 12$ with $x_0 = y_0 = z_0 = 0$ (06 Marks)
- b. Reduce the following matrix to the diagonal form
$$\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$$
 (05 Marks)
- c. Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$ to the canonical form by orthogonal transformation. (05 Marks)
